

C1 January 2014 (IAL) (MA)

$$Q1a) (2\sqrt{x})^2 = \boxed{4x}$$

$$\begin{aligned} b) \frac{5+\sqrt{7}}{2+\sqrt{7}} &= \frac{5+\sqrt{7}}{2+\sqrt{7}} \times \frac{2-\sqrt{7}}{2-\sqrt{7}} \\ &= \frac{(5+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})} \\ &= \frac{10 - 5\sqrt{7} + 2\sqrt{7} - 7}{4 - 2\sqrt{7} + 2\sqrt{7} - 7} \\ &= \frac{3 - 3\sqrt{7}}{-3} \\ &= -1 + \sqrt{7} \\ &= \boxed{\sqrt{7} - 1} \end{aligned}$$

$$Q2) y = 2x^2 - \frac{4}{\sqrt{x}} + 1, x > 0$$

$$y = 2x^2 - 4x^{-\frac{1}{2}} + 1$$

$$\begin{aligned} a) \frac{dy}{dx} &= 4x + 2x^{-\frac{3}{2}} \\ &= 4x + \frac{2}{x^{\frac{3}{2}}} \\ &= \boxed{4x + \frac{2}{x\sqrt{x}}} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{d^2y}{dx^2} &= 4 - 3x^{-\frac{5}{2}} \\ &= 4 - \frac{3}{x^{\frac{5}{2}}} \end{aligned}$$

$$= \boxed{4 - \frac{3}{x^2\sqrt{x}}}$$

Q3)

$$x - 2y - 1 = 0 \Rightarrow x = 2y + 1 \quad \textcircled{1}$$

$$x^2 + 4y^2 - 10x + 9 = 0 \Rightarrow x^2 + 4y^2 - 10x + 9 = 0 \quad \textcircled{2}$$

Substitute ① into ②:

$$(2y+1)^2 + 4y^2 - 10(2y+1) + 9 = 0$$

$$(2y+1)(2y+1) + 4y^2 - 10(2y+1) + 9 = 0$$

$$4y^2 + 4y + 1 + 4y^2 - 20y - 10 + 9 = 0$$

$$8y^2 - 16y = 0$$

$$y^2 - 2y = 0$$

$$y(y-2) = 0$$

Either $y=0$ or $y=2$

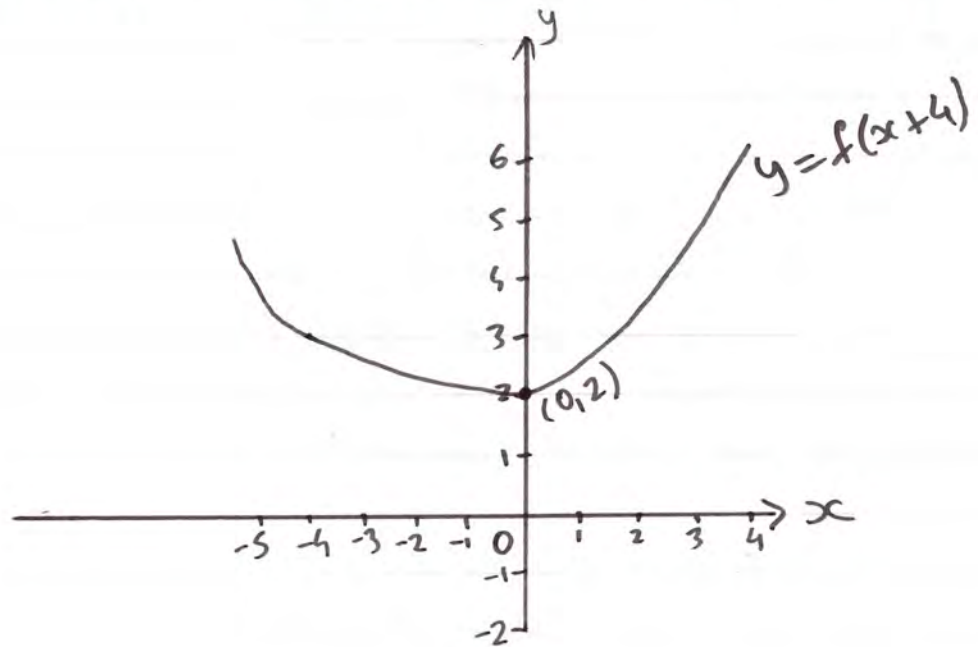
Substitute into ① for x :

$$\text{When } y=0, x = 2(0) + 1 = \underline{1}$$

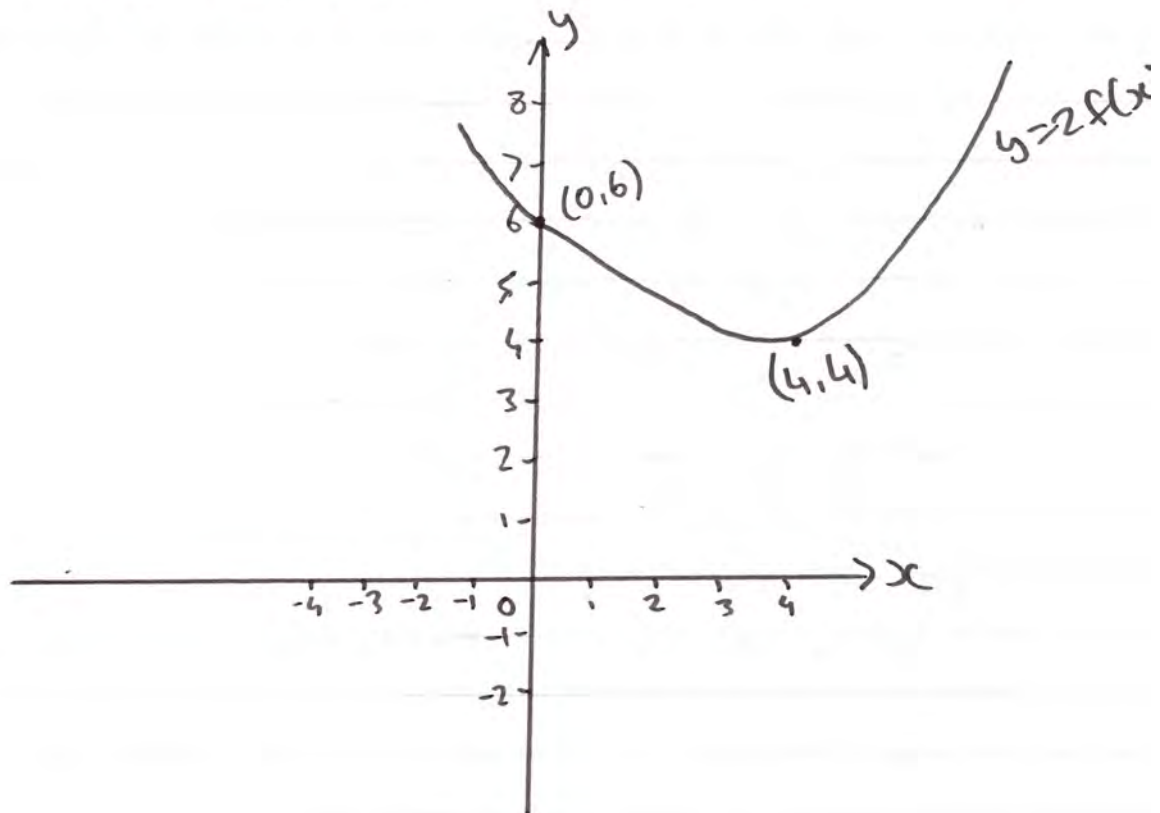
$$\text{When } y=2, x = 2(2) + 1 = \underline{5}$$

Solution set: $\boxed{x=1, y=0 \text{ and } x=5, y=2}$

Q4a) $y = f(x+4)$ - transformation horizontally of



b) $y = 2f(x)$ - transformation vertically of $\times 2$:



$$Q5) \sum_{r=1}^n a_r = 12 + 4n^2$$

$$\begin{aligned} a) \sum_{r=1}^5 a_r &= 12 + 4(5)^2 \\ &= 12 + 100 \\ &= 112 \end{aligned}$$

$$\begin{aligned} b) \sum_{r=1}^6 a_r &= 12 + 4(6)^2 \\ &= 12 + 144 \\ &= 156 \end{aligned}$$

$$a_6 = \sum_{r=1}^6 a_r - \sum_{r=1}^5 a_r$$

$$a_6 = 156 - 112$$

$$a_6 = 44$$

Q6a) l_n has equation $2y = 3x + 7$

$$i) y = \frac{3}{2}x + \frac{7}{2}$$

$$\text{Gradient of } l_1 = \frac{3}{2}$$

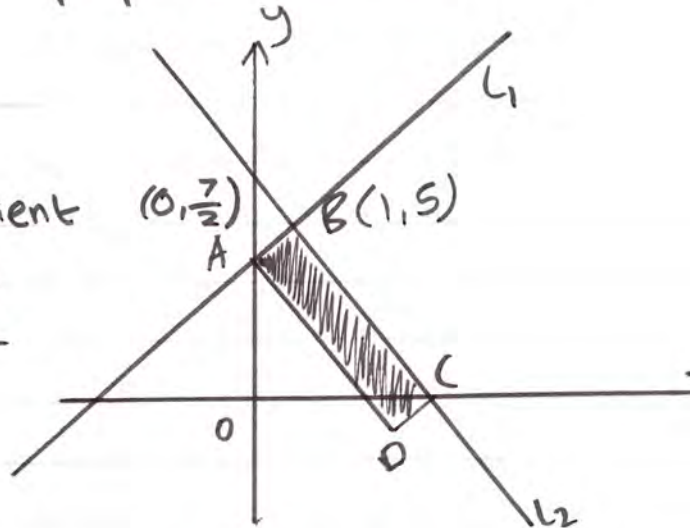
ii) when l_1 cuts the y -axis, $x=0$

$$\text{So, } y = \frac{7}{2} \Rightarrow A \text{ is at } \left(0, \frac{7}{2}\right)$$

b) l_2 intersects l_1 at $B(1,5)$ and crosses the x -axis at

From diagram, l_1 is perpendicular to l_2 ,
Since $\widehat{ABC} = 90^\circ$.

If the gradient of $l_1 = \frac{3}{2}$, then the gradient of $l_2 = -\frac{2}{3}$



Equation of l_2 :

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{2}{3}(x - 1) \leftarrow$$

$$3(y - 5) = -2(x - 1)$$

$$3y - 15 = -2x + 2$$

$$3y + 2x - 17 = 0$$

$$\boxed{2x + 3y - 17 = 0}$$

Using $B(1,5)$ and $m = -\frac{2}{3}$

c) When l_2 cuts the x -axis, $y = 0$

$$\text{So, } 2x + 3(0) - 17 = 0$$

$$2x = 17$$

$$x = \frac{17}{2} \Rightarrow C \text{ is at } \left(\frac{17}{2}, 0\right)$$

$$\begin{aligned}
 \text{Length } AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(1 - 0)^2 + (5 - \frac{7}{2})^2} \\
 &= \sqrt{1^2 + (\frac{3}{2})^2} \\
 &= \sqrt{1 + \frac{9}{4}} \\
 &= \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{\sqrt{4}} = \frac{\sqrt{13}}{2} \\
 &= \frac{\sqrt{13}}{2} \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 \text{Length } BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(\frac{17}{2} - 1)^2 + (0 - 5)^2} \\
 &= \sqrt{(\frac{15}{2})^2 + (-5)^2} \\
 &= \sqrt{\frac{225}{4} + 25} \\
 &= \sqrt{\frac{325}{4}} = \sqrt{\frac{325}{4}} = \frac{\sqrt{325}}{\sqrt{4}} = \frac{\sqrt{25 \cdot 13}}{2} \\
 &= \frac{5\sqrt{13}}{2} \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area } ABCD &= bh = (AB)(BC) \\
 &= \left(\frac{\sqrt{13}}{2}\right) \left(\frac{5\sqrt{13}}{2}\right)
 \end{aligned}$$

$$\boxed{= \frac{65}{4} \text{ units}^2}$$

Q7) First term, $a = \text{£}14\,000$
Common difference, $d = \text{£}1500$

a) $U_n = a + (n-1)d$

$$\begin{aligned} U_9 &= 14000 + (9-1)1500 \\ &= 14000 + 12000 \\ &= \boxed{\text{£}26\,000} \end{aligned}$$

b) $S_n = \frac{n}{2} (2a + (n-1)d)$

$$\begin{aligned} S_9 &= \frac{9}{2} (2(14000) + (9-1)1500) \\ &= \frac{9}{2} (28000 + 12000) \\ &= \frac{9}{2} \times 40000 \\ &= \boxed{\text{£}180\,000} \end{aligned}$$

c) For Anna, first term, $a = \text{£}A$
Common difference, $d = \text{£}1000$

Max. salary of $\text{£}26000$ reached in year 10

$$\therefore U_{10} = 26000$$

$$U_n = a + (n-1)d$$

$$U_{10} = A + (10-1)1000$$

$$26\,000 = A + 9000$$

$$\therefore \underline{A = \pounds 17\,000}$$

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_{10} = \frac{10}{2} (2(17\,000) + (10-1)1000)$$

$$= 5(34\,000 + 9000)$$

$$= 5(43\,000)$$

$$= \underline{\pounds 215\,000}$$

For Shelim, $S_9 = \pounds 180,000$

Since his salary does not go above $\pounds 26\,000$ a year, he will receive $\pounds 26\,000$ in the 10th year, making his total:

$$S_{10} = S_9 + 26\,000$$

$$S_{10} = 180\,000 + 26\,000$$

$$= \underline{\pounds 206\,000}$$

Difference between Shelim and Anna:

$$= 215\,000 - 206\,000$$

$$= \boxed{\pounds 9\,000}$$

\therefore Anna earns $\pounds 9\,000$ more in the first 10 years

Q 8) $2x^2 + 2kx + (k+2) = 0$ has two distinct real roots.

a) If a quadratic has two distinct real roots, then the discriminant is greater than 0.

$$\therefore b^2 - 4ac > 0$$

$$(2k)^2 - (4)(2)(k+2) > 0$$

$$4k^2 - 8(k+2) > 0$$

$$4k^2 - 8k - 16 > 0$$

$$\therefore \boxed{k^2 - 2k - 4 > 0}$$

b) For $k^2 - 2k - 4 = 0$

~~$$k^2 - 2k - 4 = 0$$~~

$$(k-1)^2 - 1 - 4 = 0$$

$$(k-1)^2 - 1 - 4 = 0$$

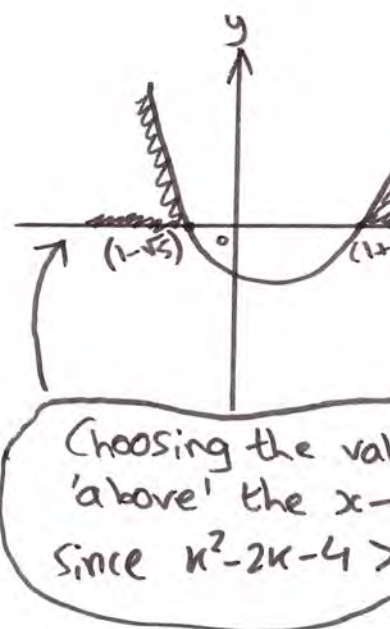
$$(k-1)^2 = 5$$

$$k-1 = \pm\sqrt{5}$$

$$k = 1 \pm \sqrt{5}$$

Set of possible values of k :

$$\boxed{k < 1 - \sqrt{5} \text{ or } k > 1 + \sqrt{5}}$$



$$\text{Q9a) } f'(x) = (x-2)(3x+4)$$

$$f'(x) = 3x^2 + 4x - 6x - 8$$

$$f'(x) = 3x^2 - 2x - 8$$

$$f(x) = \int (3x^2 - 2x - 8) dx$$

$$f(x) = \frac{3x^3}{3} - \frac{2x^2}{2} - 8x + C$$

$$\underline{f(x) = x^3 - x^2 - 8x + C}$$

Since $f(x)$ passes through $(3, 6)$, substitute in $x=3$ and $f(x)=6$:

$$6 = 3^3 - 3^2 - 8(3) + C$$

$$6 = 27 - 9 - 24 + C$$

$$6 = -6 + C$$

$$\therefore \underline{C = 12}$$

$$\boxed{f(x) = x^3 - x^2 - 8x + 12}$$

$$\text{b) } f(x) = (x-2)^2(x+p)$$

$$f(x) = (x-2)(x-2)(x+p)$$

$$f(x) = (x^2 - 4x + 4)(x+p)$$

$$x^3 - x^2 - 8x + 12 = (x^2 - 4x + 4)(x + 3)$$

$$\therefore \boxed{p=3}$$

$$x^3 - x^2 - 8x + 12 = \underline{(x-2)^2(x+3)}$$

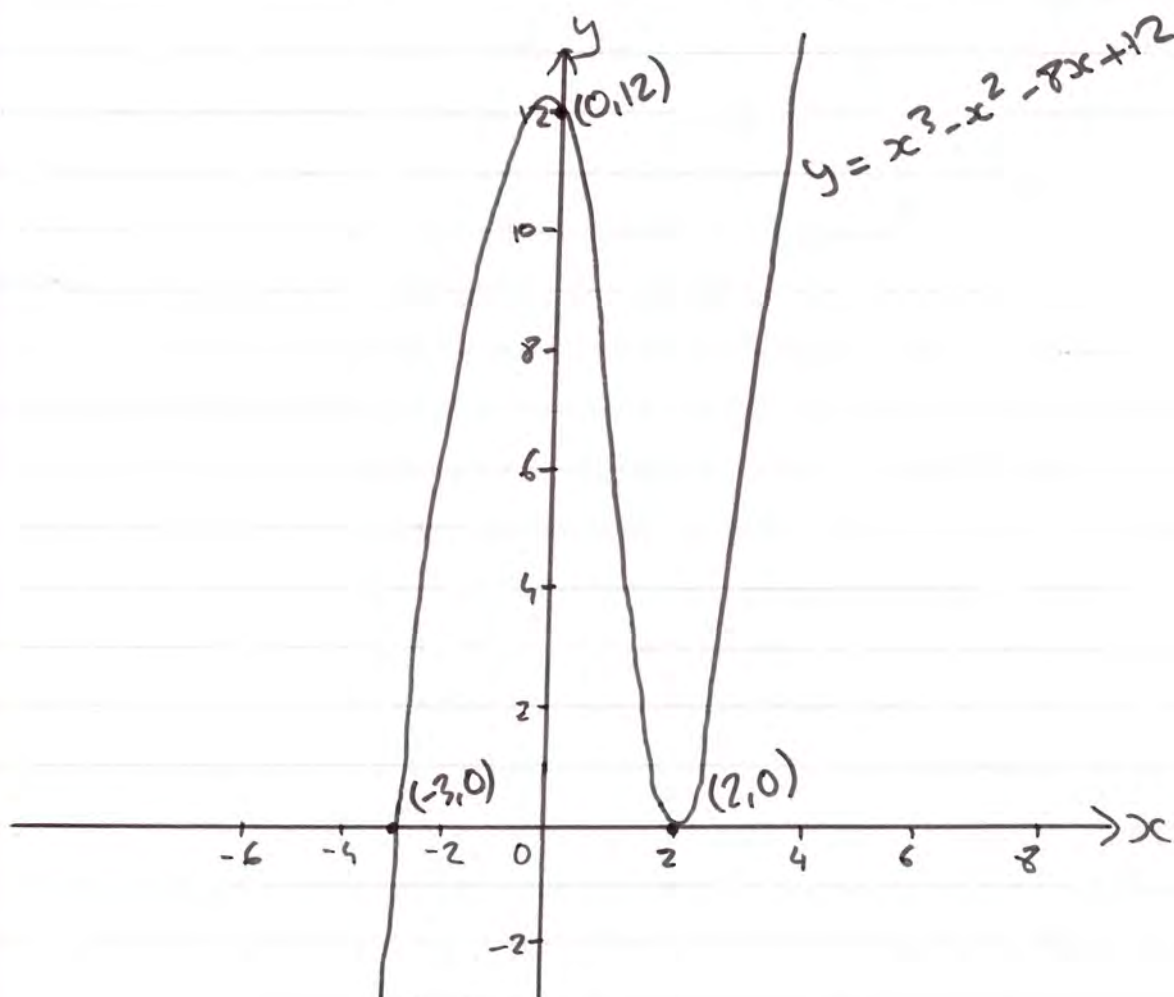
c) $y = f(x) = x^3 - x^2 - 8x + 12$

When $x=0, y=12$

When $y=0, x^3 - x^2 - 8x + 12 = 0$

$$(x-2)^2(x+3) = 0$$

Either $x=2$ or $x=2$ or $x=-3$



$$Q10a) \quad y = x^3 - 2x^2 - x + 3$$

$$\frac{dy}{dx} = \underline{3x^2 - 4x - 1}$$

$$\begin{aligned} \text{At } x=2, \frac{dy}{dx} &= 3(2)^2 - 4(2) - 1 \\ &= 12 - 8 - 1 \\ &= \underline{3} \end{aligned}$$

\therefore the gradient of the tangent at P is 3.

Equation of tangent: $y - y_1 = m(x - x_1)$

Using P(2,1)
and $m=3$

$$y - 1 = 3(x - 2)$$

$$y - 1 = 3x - 6$$

$$\boxed{y = 3x - 5}$$

b) If the tangent at Q is parallel to the tangent at P, then the gradients of both are

$$\therefore \frac{dy}{dx} = 3 = 3x^2 - 4x - 1$$

$$3x^2 - 4x - 1 = 3$$

$$3x^2 - 4x - 4 = 0$$

$$(3x+2)(x-2) = 0$$

$$\underline{\text{Either } x = -\frac{2}{3} \text{ or } x = 2}$$

We already know that when $x=2$,
 $y=1$ and this is point P.

So, when $x = -\frac{2}{3}$, this is point Q.

$$x = -\frac{2}{3}, \quad y = x^3 - 2x^2 - x + 3$$

$$y = \left(-\frac{2}{3}\right)^3 - 2\left(-\frac{2}{3}\right)^2 - \left(-\frac{2}{3}\right) + 3$$

$$y = \left(-\frac{8}{27}\right) - 2\left(\frac{4}{9}\right) + \frac{2}{3} + 3$$

$$y = -\frac{8}{27} - \frac{8}{9} + \frac{2}{3} + 3$$

$$y = -\frac{8}{27} - \frac{24}{27} + \frac{18}{27} + \frac{81}{27}$$

$$\underline{y = \frac{67}{27}}$$

\therefore Q has coordinates

$$\boxed{\left(-\frac{2}{3}, \frac{67}{27}\right)}$$